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ELEMENTARY VECTOR ANALYSIS

WITH APPLICATION TO
GEOMETRY AND PHYSICS

BY

C. E. WEATHERBURN
M.A. (CAMB.), D.SC. (SYDNEY)
LECTURER IN MATHEMATICS AND THEORETICAL PHYSICS, ORMOND COLLEGE
UNIVERSITY OF MELBOURNE
UNIVERSITY EVENING LECTURER IN PURE MATHEMATICS

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TO

JOHN HENRY MICHELL, F.R.S.

WHOSE HELP AND ADVICE
HAVE BEEN AT ALL TIMES MOST WILLINGLY GIVEN,
AND TO WHOSE INSPIRATION
THE WRITING OF THE FOLLOWING PAGES WAS LARGELY DUE,
THIS BOOK IS GRATEFULLY ASCRIBED.

PREFACE

My object in writing this book was to provide a simple exposition of elementary Vector Analysis, and to show how it may be employed with advantage in Geometry and Mechanics. It was thought unnecessary, in the present volume, to enter upon the more advanced parts of the subject, built upon the ideas of gradient, curl and divergence. Vector algebra and the differentiation of vectors with respect to one scalar variable furnish a powerful instrument even for the higher parts of dynamics.

The work does not claim to be a complete text-book in either Geometry or Mechanics, though a good deal of ground is covered in both subjects. The use of vector analysis in the former is abundantly illustrated by the treatment of the straight line, the plane, the sphere and the twisted curve, which are dealt with as fully as in most elementary books, and a good deal more concisely. In Mechanics I have explained and proved all the important elementary principles. The equations of equilibrium for a rigid body are deduced from the equations of motion. This is contrary to the ordinary practice and, of course, is not recommended for young beginners. But for a student who is able to read this volume, it is certainly desirable to show that the principles of statics are only particular cases of the dynamical ones, and that the long line of argument followed by text-books in Statics, to prove the theorems about moments, parallel forces, couples and the equilibrium of bodies, is really unnecessary. All these theorems are immediately deducible from the equations of motion of a rigid body, as shown in Chapter VIII.

Another departure from the ordinary practice has been made in connection with the theory of *centroids*. Most students gain

their introduction to centroids through centre of gravity. But at a later stage they should learn that centre of gravity is only a particular case of centroids, and that a presentation of the subject may be given which includes all cases. Arts. 9-11 were written with this object in view. It is because most students regard centre of gravity as the very essence of centroid that we continually meet such expressions as "centre of gravity of an area" or "centre of gravity of a cross section." Centroids of area and volume exist in their own right, and are quite independent of mass and weight.

In treating the geometry of the straight line, plane and sphere, my object is primarily to explain the vector method and notation, and *not* to show their superiority over other methods. The reader must decide for himself which is preferable. In connection with twisted curves, the use of vectors seems decidedly advantageous. In the case of the plane and the sphere it is chiefly brevity of expression that is gained; though a comparison of the geometrical work in Chapters III. and IV. with the corresponding theory given in books on analytical geometry will be instructive to the reader. *Vectors were, however, not designed for use in elementary plane geometry*; and any two-dimensional geometry in the sequel is introduced merely by way of easy illustration of the vector method. Vector analysis is intended essentially for three-dimensional calculations; and its greatest service is rendered in the domains of mechanics and mathematical physics.

After much consideration I decided to employ the dot and cross notation for products of vectors. This has always appeared to me the most convenient, particularly for the treatment of the linear vector function.

During the preparation of this book I have been greatly indebted to Mr. J. H. Michell, M.A., F.R.S., who read the MS. and proof sheets, and made many valuable suggestions that have been incorporated in the work. I was allowed free use of his honour lectures in Mixed Mathematics, Part I., at Melbourne University; and it was his manner of treating the subject that first led me to undertake the study of Vector Analysis. His interest and encouragement in the writing of this book have been largely responsible for its final appearance.

My thanks are also due to Prof. W. P. Milne, the editor of this series, who also read the MS. and made many excellent suggestions which I was glad to adopt. Acting on his advice, I added the Historical Introduction, which will prove interesting to many readers. I am also indebted to Mr. D. K. Picken, M.A., Master of Ormond College, for certain introductory ideas, which have to some extent influenced my presentation of the subject. And I take this opportunity of thanking Mr. Ian W. Wark, B.Sc., of Ormond College, who generously undertook the task of verifying the exercises to each chapter, and furnishing answers where necessary. I am also grateful to my college friend, Dr. T. M. MacRobert, of Glasgow University, who kindly offered to revise the final proofs.

As to other literature, I have elsewhere * acknowledged my great indebtedness to E. B. Wilson's *Vector Analysis*, which was my early instructor in the subject; and during the writing of the following pages I was influenced both consciously and unconsciously by Professor Wilson's book. Coffin's *Vector Analysis*, another American book, has also been frequently consulted by the author.

In conclusion. I wish to thank the Publishers for their unfailing courtesy, and the Printers for the excellence of their work.

C. E. WEATHERBURN.

Ormond College,
University of Melbourne,
April, 1920.

* "A plea for a more general use of Vector Analysis in Applied Mathematics," *Math. Gazette*, Jan. 1917.

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